A Method for Evaluating the Magnetic Field Homogeneity of a Radiofrequency Coil by Its Field Histogram

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The magnetic field homogeneity of a radiofrequency coil is very important in both magnetic resonance imaging and spectroscopy. In this report, a method is proposed for quantitatively evaluating the RF magnetic field homogeneity from its histogram, which is obtained by either experimental measurement or theoretical calculation. The experimental histogram and theoretical histogram can be compared directly to verify the theoretical findings. The RF field homogeneities of the bird-cage coil, slotted-tube resonator, cosine wire coil, and a new radial plate coil design were evaluated using this method. The results showed that the experimental histograms and the corresponding theoretical histograms are consistent. This method provides an easy and sensitive way of evaluating the magnetic field homogeneity and facilitates the design and evaluation of new RF coil configurations.

The homogeneity of the radio frequency magnetic or $B_1$ field is an important parameter for determining the quality of an RF coil or an RF coil design (1, 2). In optimizing a coil design, the homogeneity of the theoretically calculated $B_1$ field needs to be evaluated and compared between various coil configurations. Since the $B_1$ field distribution can vary greatly for different coil geometries, it is difficult to make a quantitative judgment of the field homogeneity directly from the field map alone. Furthermore, the $B_1$ field homogeneity of the optimized coil configuration should be evaluated experimentally to verify the results of the theoretical calculation. Therefore, in order to make such comparisons, we sought to develop a method which would characterize the global or overall homogeneity of the $B_1$ field.

Typically, the $B_1$ field is mapped using point-by-point measurement with a small pick-up coil (3) or a small water sample (4). This technique produces the spatial information about the field, but is time consuming and subject to errors (5). Alternatively, various methods of imaging a uniform sample have been used to determine the $B_1$ field map (2, 6–8). More recently, the in vivo $B_1$ distribution has been generated from the ratio of the intensities of the image from spin and stimulated echoes (9). Nevertheless, direct comparison of the calculated and the experimental $B_1$ field maps measured by these methods still remains a difficult task.

Several investigations have shown that Fourier transformation of the signal intensity as a function of flip angle produces the distribution of nutation frequency which is proportional to $B_1$ magnitude (10–12). In this paper, we will show that this procedure is, in fact, generating a histogram of the $B_1$ field intensity, which is a direct measure of the overall $B_1$ field homogeneity. A histogram from the calculated $B_1$ field can also be easily generated for direct comparison with the experimental results. Thus, the judgment on the $B_1$ field homogeneity can be easily made by inspection of its histograms. This technique, which we will refer to as the field histogram method hereafter, provides an easy and sensitive way to evaluate the $B_1$ field homogeneity of an RF coil.

For illustration purposes, we applied the field histogram method to three types of commonly used RF coils: the bird-cage coil (BCC) (13), the slotted-tube resonator (STR) (14), and the cosine wire coil (CWC) (15). These coil configurations are all believed to produce a homogenous $B_1$ field. We also present a new radial plate coil (RPC) configuration designed using the field histogram method. The configurations of these coils are shown in Fig. 1.

THEORY AND METHODS

In general the histogram of a magnetic field distribution, $B_1$ ($\chi_1$), is prepared by uniformly sampling the field magnitude, $B_1$, over the region of interest. The histogram of the field, $H(B_1)$, is then plotted as a function of $N(B_1)$ versus $B_1$.

$$H(B_1) = N(B_1), \quad [1]$$

where $N(B_1)$ is the number of the sampling points which have the same field magnitude. Equation [1] is the conventional definition of the histogram of the magnetic field distribution.
To generate an experimental histogram of the $B_1$ field, the signal intensity versus RF pulse width ($t_p$) of a single pulse sequence is measured. As a starting point, we ignore the influences of relaxation, static magnetic field inhomogeneity, and probe electronic properties. Thus, in a two-dimensional case, the induced signal from the sample is given by (12)

$$S(t_p) = K \int_A \rho(x, y) B_1(x, y) \sin[\omega_1(x, y) t_p] \, dx \, dy,$$  \hspace{1cm} [2]

where $t_p$ is the duration of the RF pulse, $\rho(x, y)$ is the spin-density function which is set to unity for an uniform sample, $B_1(x, y)$ is the RF magnetic field distribution, $K$ is a proportionality constant, $A$ is the sample area, and $\omega_1(x, y)$ is the nutation angular frequency given by

$$\omega_1(x, y) = \gamma B_1(x, y),$$  \hspace{1cm} [3]

where $\gamma$ is the magnetogyric ratio. Fourier transformation of Eq. [2] with respect to $t_p$ yields a $B_1$ field histogram [$I(\omega_1)$]:

$$I(\omega_1) = K' \int_A B_1(x, y) dx \, dy \int \sin[\omega_1(x, y) t_p] e^{-i\omega_1 t_p} \, dt_p,$$  \hspace{1cm} [4]
The formulation of $I(\omega_1)$ does not directly show the similarity to the conventional histogram definition in Eq. [1]. It can be shown, however, that $I(\omega_1)$ is directly proportional to the histogram of the $B_1$ field. This becomes evident by using the Dirac delta function (16)

$$\delta[\omega_1(x,y) - \omega_1'] = C \int \sin[\omega_1(x,y)t_p]e^{-j\omega_1't_p}dt_p,$$

where $C$ is the normalization constant; thus

$$I(\omega_1') = K' \int_A B_1(x,y)\delta[\omega_1(x,y) - \omega_1'] dx dy.$$  [5]

Rewriting Eq. [5] in the $B_1$ domain,

$$I(B_1') = K' \gamma \int_A B_1(x,y)\delta[\omega_1(x,y) - \omega_1'] dy$$

$$= K' \gamma B_1' N(B_1'),$$  [6]

where $B_1' = \omega_1'/\gamma$, $N(B_1')$ is the number of the points in the area $A$ having the same value of $B_1'$, and $K'$ has absorbed the Fourier-transformation normalization constant.

Comparison of Eqs. [1] and [6] shows that Fourier transformation of $S(t_p)$ creates an equivalent form of the histogram of the $B_1$ field or the so-called $B_1$-weighted histogram. The signature of this histogram is a direct measure of the $B_1$ field homogeneity.

In addition to the experimental histogram, the theoretical $B_1$-weighted histogram can also be created using the definition of Eq. [6]. After the $B_1$ field is calculated, the field is sampled at equally spaced grid points inside the sample area of the coil. The density of the grid points is chosen to characterize the field distribution accurately. The smaller the field gradient, the smaller the number of points needed to describe it. According to Eq. [6], the histogram is obtained by sequentially counting the number of grid points with the same $B_1$ value $[N(B_1')]$. The histogram is presented as a plot of $B_1$-weighted population $[B_1 \times N(B_1')]$ versus $B_1$. Note that the definition of the histogram here is different from its general meaning, which usually implies the relationship of population $N(B_1)$ vs $B_1$. To define the histogram this way makes it possible to directly compare the theoretically calculated histogram with the experimental histogram.

Next, the histogram is normalized to compare the $B_1$ field homogeneity of the coils with different field strengths and geometrical configurations. The horizontal axis of the normalized histogram is defined as the percentage field deviation (PFD) with respect to the coil's center magnetic field, $B_{1c}$,

$$PFD = \frac{B_1}{B_{1c}} \times 100.$$

The vertical axis is the normalized population of deviation (NPD):

$$\text{NPD} = \frac{I(\text{PFD})}{\int_A I(\text{PFD}) d\text{PFD}}.$$  [8]

Attention should be given to the resolution of the calculated histogram. Since the numerical $B_1$ values are usually eight digits or longer, very few values can be found to be identical. In order to make comparisons among the $B_1$ values, they first must be rounded off to a certain digit which subsequently defines the resolution of the histogram. The choice of resolution is not entirely arbitrary. If the resolution is too fine, such that the total number of data points along the horizontal axis of the histogram is close to or exceeds the total number of grid points, artificial noise will be generated in the histogram. This occurs from the fact that the resolution of the histogram exceeds the resolution of the sampling of the $B_1$ field. In this case, the resolution of the histogram must be decreased.

The theoretical spatial $B_1$ distribution of RF coils was calculated using the Maxwell 2D Field Simulator software, Version 4.3 (Ansoft Corp., Pittsburgh, Pennsylvania) (17, 18), which uses finite element analysis to solve the two-dimensional Maxwell equations in the form

$$\nabla \times (1/\mu) \nabla \times \mathbf{A} = (\sigma + j\omega\epsilon)(-j\omega\mathbf{A} - \nabla\phi),$$  [9a]

$$\nabla \times \mathbf{A} = \mathbf{B},$$  [9b]

where $\sigma$ is conductivity, $\epsilon$ is permittivity, $\mu$ is permeability, $\omega$ is the input current angular frequency, $\mathbf{A}$ is the vector potential, and $\phi$ is the scalar potential. The contribution of eddy currents $(-j\omega\mathbf{A})$ and displacement currents $[j\omega(-j\omega\mathbf{A} - \nabla\phi)]$ are all considered in the calculation. Users provide the coil model geometry, $\sigma$, $\epsilon$, and $\mu$ distributions of all the objects in the region under consideration, boundary conditions, and the input current intensity and frequency to the coil. The program solves for $\mathbf{A}$. The $B_1$ field is then calculated from $\mathbf{A}$.

Each coil model has a cylindrical geometry of 40 mm diameter and infinite length. The $B_1$ field is sampled uniformly from 1824 grid points inside a semicircular area of 34 mm diameter. This region approximates one-half of the effective sample area. The coil models under investigation are shown in Figs. 2a–2d. The BCC consists of eight wires with angular spacings of 45° (19). The current intensity in each wire is assigned according to the cosine of the polar angle of the wire position. The RPC is made up of 18 pieces of radially aligned copper plates, with 9 pieces on each side of a cylindrical former arranged at 10° intervals. The nine copper plates on each side are connected to a common current source. The input current on each side is assigned with equal intensity, but opposite in phase. The STR has a window.
FIG. 2. The contour plots of the calculated $R_2$ field and the corresponding histograms (right) for the (a) bird-cage coil (BCC), (b) radial plate coil (RPC), (c) slotted-tube resonator (STR), and (d) cosine wire coil (CWC). The contour plots are normalized by assuming that the center field generates a 90° pulse. The contours are labeled with the corresponding pulse strengths. The contours in very high field regions near the wires are not drawn.
angle of 95° which produces the optimal homogeneity (20). The CWC, a design of Bolinger et al. (15), consists of 22 wires on the circumference. The wire positions are determined by geometrical construction, where intervals of equal length along the diameter of the coil are projected onto the circumference. The source current for the STR and CWC is the same as that of the RPC. In all cases, the RF field frequency is 80 MHz and identical to the experimental RF frequency.

All experiments were performed in a 1.9 T, 26 cm horizontal bore Oxford magnet interfaced to a Nicolet 1180 console. The coils for each type were all 40 mm in diameter and 75 mm in length (Fig. 1). The sample was a cylindrical disk, 34 mm in diameter and 15 mm in thickness, filled with saline solution.

For the experimental measurements, the pulse width of the single-pulse sequence was incrementally increased in either 10 or 20 μs steps. Seventy to one hundred spectra were collected on resonance. A curve of the signal intensity vs pulse width was obtained. Generally, the magnetization was rotated at least six times. For the coils with very high B1 field homogeneity, a longer pulse width was generally required. Since the experiment was carried out in the rotating frame, while the pulse was on, the signal was decaying according to the nutation time constant \( T_n \) (22), where

\[
\frac{1}{T_n} = \frac{1}{2} \left( \frac{1}{T_1} + \frac{1}{T_2} \right). \tag{10}
\]

A longer \( T_n \) broadens the experimental histogram. Therefore, the signal intensity data points were first corrected by multiplying the factor \( e^{(i/\omega T_n)} \) prior to the subsequent data processing. The experimental data were fitted to damped sinusoidal curves using the Peakfit program (Jandel Co., California), which runs off-line on an IBM PS/2 Model 55 personal computer. This fitting step avoids Fourier transformation of a truncated waveform. If the coils have poor homogeneity, the signal decays rapidly to zero with increasing pulse width. This allows Fourier transformation to be directly applied after zero filling to produce a histogram of the B1 magnetic field within the sample volume.

RESULTS AND DISCUSSION

Figure 2 shows the computer-calculated B1 fields for each coil in two different representations. The commonly used contour plots shown on the left side are very helpful in visualizing the B1 field spatial distribution. They do not provide, however, a straightforward quantitation of the field homogeneity, especially when the contours are quite different in shape. The histogram discards the details of B1 field spatial distribution, but more clearly shows the overall homogeneity. A high and narrow peak represents good field homogeneity, as indicated in Fig. 2a for the BCC, and a broad, flat “hump” represents an inhomogeneous field as demonstrated in Fig. 2d for the CWC. The total field dispersion within the entire sample region for the BCC is within ±10 PFD units, and ±5 PFD covers up to 70% of the total sample area. In contrast,
The experimental histogram (solid line) and calculated histogram (dash line) for the radial plate coil (RPC). The CWC field extends to +30 PFD with two broad peaks of relatively homogeneous regions separated by about 15 PFD. In general, the histogram with the smaller dispersion of PFD represents the better field homogeneity. Upon comparison of all the histograms in Fig. 2, the homogeneity for the RPC is the closest to that of the BCC and significantly better than that of the STR. The field for the CWC exhibits poor homogeneity. The field homogeneity shown by the histogram makes the judgment of the quality of the field homogeneity easy and straightforward. Thus, the term “field homogeneity” can be precisely and quantitatively defined in terms of the histogram.

Figure 3 shows the experimental histograms of the BCC, RPC, STR, and CWC. The experimental histogram for the BCC displays a higher amplitude and narrower distribution, indicating superior field homogeneity as demonstrated theoretically in Fig. 2. The homogeneity for RPC is significantly improved as compared with the STR. The experimental histogram of the CWC reproduces the calculated histogram reasonably well and indicates poor field homogeneity. The experimental results correlate closely with the theoretical calculations. In Fig. 4, both calculated and experimental histograms for the RPC are shown on the same scale. The experimental histogram agrees with the calculated histogram qualitatively, but it is wider and lower in amplitude. Similar behavior can also be seen for other coils. This discrepancy can be caused by a number of factors. First, the experimental coils have finite length and the corresponding histograms were obtained from a three-dimensional sample instead of a two-dimensional cross-section surface. The field variation

FIG. 4. The experimental histogram (solid line) and calculated histogram (dash line) for the radial plate coil (RPC).

a

b

FIG. 5. (a) The histogram for the slotted-tube resonator (STR) with a 70° window angle. The two peaks are separated by 6 PFD. (b) Plots of grid points inside the coil with \( B_1 \) values within the regions of k3.5 PFD about peak \( P_1 \) and k2.5 PFD about peak \( P_2 \) in the histogram, which corresponds to 18 and 13% of the total coil area, respectively. \( P_1 \) covers the coil’s center area and \( P_2 \) covers the areas near the windows. The semicircles show the sampling areas.
along the \( Z \) direction could significantly broaden the experimental histogram. Second, the experimental histogram has much lower resolution because \( t_p \) was limited by the longest possible pulse width. This will also attenuate the sharp component of the histogram more than the broader component. For example, the width of the center sharp component in the calculated histogram is only about 5 Hz, and the broad component is about 20 Hz. If the \( t_p \) lasts 1 ms, the 5 Hz component is attenuated approximately four times more than that of the 20 Hz component. This effect may be reduced by adjusting the transmitter power output to vary the \( \phi \), instead of changing the pulse width, or by adjusting both \( t_p \) and power output, if it is instrumentally feasible. Third, there always exist certain minor defects in the coil construction as compared with the ideal computer model. The theoretically calculated histogram provides an optimal limit of the coil configuration. Hence, the experimental histogram can be used not only to examine the homogeneity of a theoretical coil design, but also to check the quality of the constructed coil. Despite this discrepancy, the experimental histogram can still be compared rigorously with the calculated one, as long as the experimental broadening effects are taken into consideration. Since the experimental histograms are broadened in the same way for each coil configuration, it should not affect the assessment of the field homogeneity by comparing the experimental histograms of different coils.

The histogram and the field map may be used to dissect the components of the \( B_1 \) field distribution. To illustrate this, note how the window angle of STR affects the \( B_1 \) field homogeneity. For instance, consider the fields of two coils having window angles of 70° and 95°, respectively; the histogram for the 70° window configuration displays a bimodal distribution as shown in Fig. 5a. There are two relatively homogeneous regions with different magnetic field intensities. In order to describe where these regions are inside the coil, we plotted out the field maps separately showing the grid points with \( B_1 \) values within each peak (Fig. 5b). It reveals that peak \( P_1 \) at the lower field corresponds to the center area of the coil. This field component covers less than 18% of the total coil area within the peak width of \( \pm 3.5 \) PFD. The second peak \( (P_2) \), deviating from the center field strength by 6 PFD, covers 13% of the total coil area within \( \pm 2.5 \) PFD peak width. Peak \( P_2 \) corresponds to the higher field region near the windows. This observation indicates that it is necessary to increase the window angles to make these two distinct field regions merge together. In Fig. 6, the histogram of the optimal coil window angle of 95° is shown. The two peaks merge into a single sharp peak with a width of \( \pm 7 \) PFD, covering almost 70% of the total coil area. The information from this kind of analysis is very instructive and provides insight into the relationship between the electrical and the geometrical configuration of the coil and \( B_1 \) field distribution.

The procedure described here can reduce or eliminate the trial and error processes used in coil design.

It is well known that a uniform transverse magnetic field in a long cylinder may be produced by a current distribution of the cosine of the azimuthal angle \( \Phi \). The total current of the BCC approaches the ideal cosine current distribution. It is interesting to see from the field-contour plot in Fig. 2d that the current distribution of the CWC deviates from the cosine current distribution, even though the conductor arrangement has cosine characteristics. This occurs because RF current is not distributed evenly in the wires, although the wires are supplied with the RF power from a common terminal. As a result, one cannot expect the RF current to have a similar distribution to the conductor arrangement. From the contour plot in Fig. 2c for the STR, we can directly deduce that the current is concentrated in the vicinity of the four edges, which is the main source of its field inhomogeneity. In our RPC design, we successfully redistributed the current into the center region of the conductor arcs by using discrete plates oriented in the radial direction, resulting in a more homogenous RF field. The optimal window angle for the radial plate configuration is found to be 100°. Although the RPC does not achieve the homogeneity of the BCC, it is much easier to construct.

The field histogram method provides an excellent way to theoretically and experimentally assess the homogeneity of the \( B_1 \) field. One difficulty of the method is that the generation of the experimental histogram requires a RF pulse that is
long enough to rotate the magnetization five to six times. This could be very demanding on the transmitter power for larger coils. However, one can always construct a scaled-down model coil to obtain the experimental histogram if it is desired.

CONCLUSION

Sensitive and precise theoretical and experimental histograms of the $B_1$ magnetic field allow direct comparison of coil homogeneity. The field histogram method permits the simple and straightforward quantitative evaluation of the $B_1$ field homogeneity, and the easy development and modeling of new coil designs. Using this technique, a radial plate coil configuration with excellent $B_1$ homogeneity has been developed. The coil is easy to construct and offers significant improvement in field homogeneity over the standard slotted tube resonator design.

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REFERENCES